SOLAR IRRADIANCE CONVERSION MODELS*

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1. Introduction

1.1. Problem definition
There are three basic levels of modeling which might be considered when estimating, on an hourly basis, the irradiance received by a sloping surface in a given location, depending upon the data available to the user. These three types of models are as follows.

(1) Conversion or transposition models that use available direct and horizontal global or diffuse hourly irradiance records to compute hourly global irradiance values on tilted planes [1 - 7].

(2) Models which use hourly global radiation as input to generate the diffuse and direct components needed to obtain values for tilted planes [1, 8, 9].

(3) Models which generate global and/or direct irradiance from other quantities, such as percent sunshine, cloud cover, satellite imagery, etc. [10 - 12].

This paper focuses on the first type of model. It is therefore assumed that global and direct hourly data are either available (which is the case for over 40 locations in the U.S.A. [13, 14]) or accurately modeled.

The need for adequate modeling in this area is crucial for both HVAC and PV system designers, as instantaneous computational errors may reach 200 W m$^{-2}$ under certain circumstances. A recent study by Menicucci [15], showed that transposition models could be the largest source of error, depending on the model used, associated with the simulation of PV systems.

1.2. Conversion models: input parameters, model components
All the hourly transposition models presented herein are equivalent in the sense that they rely on the same set of input parameters. Besides global irradiance $G_{h}$, direct normal irradiance $I_{n}$, (or diffuse irradiance $D_{h}$), the other inputs needed are: the location’s latitude $\phi$, the plane slope considered $s$, and azimuth $\gamma$, the julian date $n$, the true solar time $t_s$, and the surround-

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ing ground albedo \( A \). The output of these models consists of the global irradiance received by the tilted plane.

Most present models also divide the energy received into three components, which are computed independently. These are the direct beam, the sky-diffuse and the ground-reflected components.

Differences between models are found mainly in the computation of the sky-diffuse component. Indeed, the distribution of radiance throughout the sky is difficult to represent adequately, and the assumptions used to describe it have proved to be the main cause of error associated with these models.

The modeling of direct radiation is identical for all models. It is a straightforward problem, and is a negligible source of error, for flat plate collectors, if accurate input data are used.

The ground-reflected component may be difficult to model with precision, and large computational errors may occur under certain circumstances. However, this component carries in most instances, less weight than the two previous ones.

2. Conversion algorithms

The hourly energy \( G_c \) impinging on a tilting plane is given by

\[
G_c = I_c + D_c + R_c
\]

(1)

where \( I_c \) is the direct component, and \( D_c \) and \( R_c \) are the sky-diffuse and the ground-reflected components respectively.

2.1. Direct component

Given direct irradiance \( I \) as input, \( I_c \) is obtained from

\[
I_c = I \left\{ \max \left( 0, \cos \theta \right) \right\}
\]

(2)

where \( \theta \) is the solar incidence angle on the plane considered; \( \cos \theta \) is given by [16]

\[
\cos \theta = \sin \delta \sin \phi \cos s - \sin \delta \cos \theta \sin s \cos \gamma
+ \cos \delta \cos \phi \cos s \cos \omega + \cos \delta \sin \phi \sin s \cos \gamma \cos \omega
+ \cos \delta \sin s \sin \gamma \sin \omega
\]

(3)

where \( s, \gamma, \) and \( \phi \) are given as input, and \( \delta \) is the declination obtained from the julian date [16]. The hour angle \( \omega \) is obtained from \( t_s \) knowing that \( \omega = 180^\circ \) at 12 a.m. and decreases at the rate of \( 15^\circ \) h\(^{-1}\).

In the case of non-fixed surfaces, where \( s \) and \( \gamma \) are not known \textit{a priori}, it is still possible to evaluate \( \cos \theta \) by using the tracking characteristics (axis, rotational speed) as input. Examples are given below for specific configurations.
For the two-axis tracker the slope and incidence angle are given by

\[ s = \arccos \left( \sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega \right) \]
\[ \theta = 0 \]  
(4)

In the case of the one-axis tracker (vertical axis, azimuth tracking) the slope is known, and the incidence angle is given by

\[ \theta = \arccos \left( \sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega \right) - s \]

and for the one-axis tracker (north–south, latitude-set axis, constant rotational speed) slope and incidence are given by

\[ s = \arccos \left( \cos \omega \cos \phi \right) \]
\[ \theta = \delta \]  
(6)

whereas for the one-axis tracker (north–south axis set at slope angle \( \alpha \), constant rotational speed) slope and azimuth are given by

\[ s = \arccos \left( \cos \omega \cos \alpha \right) \]

and

\[ \gamma = \frac{\sin \omega}{|\sin \omega|} \arccos \frac{\cos \omega \sin \alpha}{\sin \left( \arccos \left( \cos \omega \cos \alpha \right) \right)} \]  
(8)

The incidence angle \( \theta \) is then obtained through eqn. (3).

For the one-axis tracker (east–west horizontal axis, altitude tracking) the slope is given by

\[ s = \arctan \left( \frac{\cos \delta \sin \phi \cos \omega - \sin \delta \cos \phi}{\sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega} \right) \]  
(9)

The incidence angle \( \theta \) is obtained from eqn. (3), knowing that \( \gamma = 0 \).

2.2. Reflected component

The classical approach to the modeling of this component [1], assumes that a constant radiance originates from every point of the ground. Based on this assumption, \( R_e \) is given by

\[ R_e = G_h A \left( 1 - \cos \theta \right) / 2 \]  
(10)

Values of 0.2 and 0.7 for \( A \) are suggested for bare ground and snow covered ground respectively [1].

Non-isotropic approaches have been proposed (e.g. [2, 7]) based on experimental data recorded at particular sites. However, published studies have not shown improvement on the isotropic assumptions for independent tests [17]. In fact, the primary concern of the user for the modeling of this component should be the determination of the appropriate albedo and of existing important differences of albedo with azimuth.
2.3. Diffuse component

The most widely used method for the calculation of this component has been to assume an isotropic distribution of radiance throughout the sky dome [1]. The diffuse radiation received by the slope is given by

$$D_{c\text{ iso}} = D_h(1 + \cos s)/2$$  \hspace{1cm} (11)

The simplicity of the method and its reasonable accuracy have been the main reasons for its widespread use. However, as energy system modeling became more refined, the need for more accurate modeling of the solar input became obvious.

The first attempt at incorporating the observed directionality of diffuse radiation consisted of assuming that this originated entirely from the solar disc according to

$$D_{c\text{ dir}} = D_h \cos \theta/\cos Z$$  \hspace{1cm} (12)

where $Z$ is the solar zenith angle. This algorithm performed generally more poorly than the isotropic.

The next step consisted of modeling the diffuse component as a combination of the two previous approaches as

$$D_c = F D_{c\text{ iso}} + (1 - F) D_{c\text{ dir}}$$  \hspace{1cm} (13)

LeQuere [18], suggested a value of 0.8 for $F$. Hay and Davies [4] took the concept a step further by formulating the degree of anisotropy $(1 - F)$ as the ratio of terrestrial direct radiation to extraterrestrial radiation. Noticeable improvement on the isotropic approach was noted. Similar, although more complex approaches by Willmott [20], and Puri et al. [5] were not found to create additional improvement [19].

Klucher [3] took a different approach by adapting the Temps and Coulson [2] clear day model to all sky conditions. This model multiplies the isotropic value by two factors reflecting the anisotropic effect of both circumsolar and horizon brightening.

$$D_c = D_{c\text{ iso}}(1 + K \sin^2(s/2)(1 + K \cos^2 \theta \sin^3 Z))$$  \hspace{1cm} (14)

As in the Hay and Davies model [4], the factor $K$, expressing the degree of anistropy, is a function of one parameter describing the amount of direct radiation received at the earth's surface.

$$K = 1 - (D_h/G_h)^2$$  \hspace{1cm} (15)

Perez et al. [6] also included the two anisotropic effects but used a systematic approach including: (1) a simple representation of the sky dome (Fig.1); (2) circumsolar and horizon anisotropy factors ($F_1$ and $F_2$) established from experimental data and varying independently with insolation condition; (3) a three-dimensional representation of the latter making full use of the input available to the user. The three dimensions are, (a) $Z$, (b) $D_h$, and (c) $\varepsilon = (I + D_h)/D_h$. The equation is given by
Fig. 1. Representation of the three radiative components seen by a tilted plane (direct, diffuse and reflected) and representation of the sky dome used in the Perez algorithm (sky radiance is respectively equal to \(L\), \(F_1L\) and \(F_2L\) for the main the circumsolar and the horizon zone).

\[
D_c = (D_{c,\text{iso}} + D_h(F_1 - 1) a(\theta) + b(s)(F_2 - 1))(1 + c(Z)(F_1 - 1)) \\
+ d(F_2 - 1))^{-1}
\]

(16)

where \(a(\theta)\) and \(b(s)\) are factors representing the solid angle covered by the circumsolar region and the horizon band, respectively, multiplied by their average incidence on the considered plane, while \(c(Z)\) and \(d\) are the equivalent of \(a(\theta)\) and \(b(s)\) for the horizontal.

3. Model performance

Four of the algorithms described above are compared. These include the isotropic, Hay, Klucher and Perez models.

Comparison is based on three types of observation: (1) observation of root mean square errors and mean bias error, generated by all models when tested against experimental data sets; (2) observation of real-time performance for typical insolation conditions, and (3) ability of models to perform adequate array design optimization.
3.1. Long term performance

The overall performance of each model is presented for four climatic environments in Fig. 2. Hourly test data sets were obtained from Albany, NY (humid continental climate), San Antonio, TX (semi-arid), Carpentras, France (mediterranean) and Trappes, France (maritime). These sets including ground-shielded radiance measurements in vertical planes facing south, north, east and west and a south facing fixed sloping plane, are respectively 3 months, 3 months, 24 months and 21 months long. Root mean square and mean bias errors have been plotted against location for each slope and orientation. Two versions of the Perez model are included: (1) a version developed from Albany data and tested independently against the other sets, and (2) a dependent version based on each site's data.

Examples of monthly variations of the algorithms' performance are plotted in Fig. 3 for two climatically distinct sites, Goodnoe Hill, WA (mari-
time) and Phoenix, AZ (arid). Test data are normal incidence global measurements (two-axis tracking) performed by SERI. The ground component was assumed to be isotropic and albedo was set equal to 0.2. The Albany version of the Perez model was used in both cases.

3.2. Real-time performance

The difference between modeled and measured irradiance impinging on test sensors in Albany has been plotted in Fig. 4 for two illustrative examples; (1) 6/12/80, clear day, west facing vertical sensor and (2) 3/2/80, partly cloudy day, 53° tilt south facing sensor.

The maximum isotropic error reaches 150 W m⁻² in both cases. This is indicative of the type of error that would have been observed throughout the day for a tracking plane.

3.3. Array design suitability

The choice of optimum array configuration is strongly influenced by climate as seen in Fig. 5 where the relative energy received by four designs (concentrator, one-axis, two-axis tracking and fixed tilt south facing) has been plotted for Albany, NY and Phoenix, AZ. Model performance in this area is summarized in Table 1. Measured energy ratios between two configurations are compared to modeled ratios. The Albany version of the Perez model was used.
Fig. 5. Monthly variations in the relative energy received using different array mounting configurations for (a) Phoenix, AZ and (b) Albany, NY. Curves (1), two-axis flat plate/concentrator; curves (2), two-axis flat plate/single-axis flat plate; curves (3), two-axis flat plate/fixed flat plate.

4. Conclusions

There exist today methods which have demonstrated a substantial performance improvement in the modeling of the diffuse irradiance component, when compared to the classical methods. It is important to note that these methods do not require any additional input.

There is still need for improvement, however, in the following areas: (1) better understanding of climate/environment effects on model performance; (2) simplification of algorithm end use (notably the Perez algorithm), and (3) adaptability of models to: (a) applications with restricted fields of view such as low concentration arrays, and (b) simulation of more spectrally sensitive materials than crystalline silicon modules.
TABLE 1

Ability of the models to simulate overall energy ratios between different PV mounting configurations for Phoenix, AZ (12 months of hourly data) and Goodloe Hills, WA (7 months of data)

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Measured ratio</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.27</td>
<td>6.9</td>
</tr>
<tr>
<td>B</td>
<td>1.38</td>
<td>1.8</td>
</tr>
<tr>
<td>C</td>
<td>1.41</td>
<td>11.4</td>
</tr>
<tr>
<td>D</td>
<td>1.37</td>
<td>8.2</td>
</tr>
</tbody>
</table>

A, Phoenix AZ, track flat plate/concentrator; B, Phoenix, AZ, track flat plate/fixed tilt; C, Goodloe Hills, WA, track flat plate/concentrator; D, Goodloe Hills WA, track flat plate/fixed tilt.

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References

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