Modeling Sky Luminance Angular Distribution for Real Sky Conditions: Experimental Evaluation of Existing Algorithms

R. Perez, J. Michalsky, and R. Seals

Introduction

Skylight and direct sunlight are the two components of natural light. The former is an extended source of spatially varied intensity, while the latter can be regarded as a point source. Recent developments in daylighting simulation tools can accurately model light penetration within complex structures and effectively complement the time-consuming and expensive scale model approach if provided with a proper description of these two light sources.

This paper is concerned with skylight. A recent paper by the authors dealt with the direct sunlight component. A proper description of the skylight source entails a knowledge of the angular distribution of skyl luminance. Standard models concerned with integrated diffuse illuminance impinging on a tilted plane are insufficient.

Many spatially continuous luminance distribution models have been proposed. In particular, the CIE standard clear, overcast, and overcast skies in this paper, we are concerned with models designed to account for changing light intensity distribution as a function of all insolation conditions from overcast through partially cloudy to clear.

Methods

The experimental database

Models are evaluated against an experimental set of data that includes more than 16,000 all-sky scans recorded in Berkeley, CA, over the period of June 1983 and December 1986. Each scan by Lawrence Berkeley Laboratories comprises 186 luminance measurements. Measurements were performed using a multi-purpose scanning photometer developed by Battelle, Pacific Northwest Laboratories, and well suited to the current research application. In addition to sky scans, we also have time-coincident direct luminance measurements.

Unfortunately, no time coincident irradiance measurements are available; however, the use of both direct luminance and diffuse luminance (through sky scan integration) allows us to adequately describe insolation conditions and to model the corresponding direct and diffuse irradiance.

The Luminance Distribution Algorithms

We selected six algorithms that are capable of modeling the variations of skyluminance distribution as a function of all insolation conditions. Before describing each algorithm, it is important to remark that each is independent from the data set.

Brungener algorithm—The Brungener algorithm was designed to model radiance rather than luminance distribution, hence it may be at a disadvantage in the present evaluation. It uses global and diffuse irradiance as input and parameterizes insolation conditions as a two-dimensional space defined by the ratio of global to extraterrestrial irradiance (i.e., clearness index, k) and the ratio of diffuse to global irradiance. The geometrical framework of the model is based on what originally proposed by Hooper, et al. The governing equation as adapted for luminance distribution is provided in the Appendix.

Harrison algorithm—This model uses two luminance distribution profiles: overcast and clear sky, respectively, \( L_{oc} \) and \( L_{sc} \). The general profile is a linear combination of the two, based upon opaque cloud cover. The analytical expression for each profile is provided in the Appendix.

Unfortunately, opaque cloud cover data were not available for Berkeley. We did try data obtained in San Francisco, CA, located 30 km away, but results were not encouraging. Much better performance was obtained by estimating the needed input from available data, as a consequence, this model may also be at a disadvantage in the present comparison. After having tried several cloud cover algorithms, the best results were obtained by substituting \( k \) to opaque cloud cover.

Kittler homogeneous sky algorithm—This physically based model was developed to calculate both absolute and relative sky patterns for varying insolation conditions parameterized using the illumination turbidity coefficient, \( T_p \). Illumination turbidity may be derived from our direct luminance data as shown in the Appendix, along with the model governing equations. Note that we use an average luminance value for the model’s normalizing constant \( C_n \). This does not influence the results of the present evaluation because all models are compared in relative terms after...
normalization to horizontal diffuse illuminance (Rule 2 below).

*Matsura Algorithm*—This model is based on the three CIE standard skies overcast, intermediate, and clear. The all-weather distribution is obtained by linear combinations of the intermediate, the overcast, and the clear sky standard models, based on the illuminance cloud ratio, $k_i$, which is used to parameterize insolation conditions. $(k_i)$ is the illuminance equivalent of $k_i$. The model's governing equation is provided in the Appendix.

Note that the author provides specific formulae to estimate the zenith luminance input to each of the CIE skies. In the present case, we used measured zenith luminance. Moreover, since all models are normalized to horizontal diffuse illuminance (as in Rule 2 below), the value of the zenith luminance input has no influence on our evaluation.

*Pettay and Smith model*—This model uses a unique formulation relating sky luminance to diffuse irradiance, with adjustable coefficients depending on insolation conditions. Five sets of coefficients are provided for insolation conditions from overcast to clear, parameterized by a cloud index, $N_c$, defined as $(1-k)/(1-k_c)$, where $k_c$ is the theoretical clear sky value of $k$. The coefficients were derived experimentally from luminance measurements at five points in the sky. The governing equation is given in the Appendix.

*ASRC CIE combination algorithm*—As for the Matsura algorithm, this model is based on the three CIE skies plus a high turbidity formulation of the CIE clear sky. The general distribution is obtained via interpolation involving two quantities: sky clearance, $\epsilon$, and sky brightness, $\Delta$. The two parameters may be derived from direct (or global) and diffuse irradiance as shown in the appendix. The interpolation scheme is based on another model that we developed to compute diffuse illuminance on a tilted plane. This scheme has been slightly modified since the preliminary algorithm by developing a more straightforward relationship between the illuminance model's coefficients and the CIE skies.

**The ground rules for model comparison**

1. **Rule 1**—The input to each model consists of global and direct (diffuse) irradiance. This is consistent with the goal of the day lighting community to exploit irradiance data to derive daylight quantities. Models that use illuminance as input (e.g., Kittler) are not at a disadvantage here, since irradiance and illuminance are linked by a deterministic luminous efficacy. In fact, these models may even have a slight advantage, since illuminance is the measured quantity while irradiance is modeled. Only the Harrison model requiring opaque cloud cover as input is at a disadvantage.

2. **Rule 2**—We are interested in each model's ability to account for the relative angular distribution of luminance in the sky, not its ability to provide absolute luminance values. This is because given a relative profile, one can always obtain the absolute values if either the following is known: the luminance at any given point in the sky or the integrated diffuse illuminance on any given plane. Both may be obtained in a more straightforward manner from irradiance input than luminance distribution. For the present, we will compare algorithms after having normalized their output to horizontal diffuse illuminance.

**Results**

Overall results are presented in Table 1. The table includes model mean bias (MBE) and root mean square error (RMSE) for all events (15,000 scans, three million data points) in the entire sky dome divided into five regions: the zenith region (60-degree elevation and greater) and three regions below 60 degrees elevation in azimuthal directions facing the sun, opposite the sun, and east–west of the sun. Note that the bias error in the zenithal region is a measure of each model's ability to relate zenith luminance and diffuse illuminance on the horizontal.

**Ideal model benchmark**—Table 1 also includes the results of an "ideal" model. This model consists of the mean skylight distributions assembled from the experimental database for each of 750 insolation conditions bins defined by the sky clearness, $\epsilon$, the sky brightness, $\Delta$, and the solar zenith angle, $Z$. This crude data-dependent model features more than 10,000 empirical coefficients (186 positions in the sky vault times 750 bins). It is not intended for use as a working model, but rather as a benchmark to estimate how much dispersion remains (RMSE) even when unbiased, insolation-dependent luminance distributions are used. In effect, this model shows the limit of accuracy achievable with the type of all-weather model considered here. The remaining RMSE would be
Table 1—Mean sky luminance (cd/m²), model mean bias, and root mean square errors as a function of sky condition and position in the sky

<table>
<thead>
<tr>
<th>All conditions</th>
<th>Mean luminance</th>
<th>Zenithal region</th>
<th>Sun-facing region</th>
<th>E/W of-sun region</th>
<th>North-of-sun region</th>
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<tr>
<td></td>
<td></td>
<td>MBE RMSE</td>
<td>MBE RMSE</td>
<td>MBE RMSE</td>
<td>MBE RMSE</td>
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<td>87 -1200</td>
<td>-10 3776</td>
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<td>247 2092</td>
<td>-489 4157</td>
<td>-228 1511</td>
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<tr>
<td>Harrison</td>
<td>197</td>
<td>2363</td>
<td>288 2045</td>
<td>-1063 4324</td>
<td>-195 1522</td>
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<td>287 2338</td>
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<td>-27 1489</td>
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<td>461 2559</td>
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<td>-250 2285</td>
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</tr>
<tr>
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<td>-204 2474</td>
<td>-310 987</td>
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</tr>
<tr>
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<td>1313</td>
<td>1199</td>
<td>1055</td>
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<tr>
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<td>108 492</td>
<td>-146 752</td>
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<tr>
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<tr>
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<td>15 396</td>
<td>-45 557</td>
<td>356 888</td>
<td>-40 513</td>
<td>-122 545</td>
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</table>

mostly attributable to one-of-kind random cloud/haze patterns that the present models cannot address.

Table 1 also illustrates model behavior for three specific insolation conditions: clear sky conditions, bright overcast, and dark overcast. In each case, models have been ranked with respect to their overall RMS error. The validation results reported in Table 1 include all solar elevations: it is interesting to note that the ranking of the models and their relative errors do not change appreciably with solar elevation.

These results are graphically illustrated in Figure 2, where model bias trends throughout the sky dome may be observed for a specific range of solar elevations (~ 45 degrees).

Discussion

Results in Table 1 show that overall the ASRC-CIE combination model exhibits the lowest RMSE in all orientations and a low MBE for all but the direction opposite the sun. The Brunger algorithm comes in second in terms of RMSE with the best performance for dark overcast conditions. This is remarkable since this model was not designed as a luminance distribution model. Following are the Harrison model (remarkable...
given the fact that we used a substitute input), the Matsuura, Kittler, and Perraudau algorithms.

It is interesting to note that the two models with the best performance rely on a two-dimensional parameterization of insolation conditions that, in effect, differentiates between the clearness of the sky (overcast vs. clear) and its brightness (thin vs. thick cloudiness). This would support the thesis that a key factor in the successful modeling of skylight distribution is an adequate parameterization of insolation conditions, as much as the model's geometrical framework. The fact that the Matsuura algorithm...
which uses the same CIE functions as the ASRC algorithm but another parameterization, performs less well, tends to confirm our assertion. Indeed, Figure 2 and Table 1 show that the clear sky performance and dark overcast sky performance of the Matsuura algorithm is very similar to that of ASRC, but that the former does not account well for bright overcast skies.

Note that a more versatile parameterization does not necessarily mean more input quantities. In fact, all models tested here use exactly the same input information.

A similar observation is made for the Kittler model which performs similarly to the ASRC and Matsuura algorithms for the extreme conditions, but does not handle the intermediate cases well. The probable reason is that the turbidity parameter ceases to be applicable as soon as skies become inhomogeneous (i.e., partly cloudy).

The main source of weakness using the Perraudeau algorithm may be the fact that this was experimentally derived from only a limited number of points in the sky dome, leading to possible distortions in other directions. The large RMSEs in the directions perpendicular and opposite to the sun are symptomatic of this.

Finally, it is very important to remark that the ideal model performance, exemplified by the mean sky model, is not considerably better than the best algorithms tested here, although possibilities for systematic improvement exist as seen from the well defined bias patterns. Also, the ideal model RMSE remains significant because of the random nature of clouds superimposed on the mean sky light distribution background. With support from the US National Science Foundation and our staff, we are in the process of developing an algorithm that will account for this random effect.

Conclusions

Our study illustrates some basic weaknesses and strengths of existing methods to model the distribution of skylight from irradiance data for real sky conditions. The best algorithms tested here, the ASRC-CIE and Brungre4 models, come close to the precision achievable by this type of model although room for systematic performance improvements exist.

We also note that the random nature of clouds, while having little impact on average skylight distribution, accounts for much short-term uncertainty, particularly in the vicinity of the sun.

The reader may want to use the results of this validation to select a model that extrapolates sky luminance spatial distribution from routine irradiance data. It must be remembered that the validation is site specific and will have to be repeated when data become available. On the other hand, to the credit of this study, it must be said that the Berkeley data feature a wide range of insolation conditions from dark overcast to very clear; that it has been shown that site dependency is not a major issue if insolation conditions are properly parameterized; and that the present test is fully independent.

Acknowledgements

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References

Nomenclature

\begin{align*}
A & \quad \text{normal illuminance from any sky element, normalized to } E_{\text{n}} \\
sc, \, a, \, \gamma & \quad \text{weighting factors used in Matsuura model} \\
\alpha, \, \beta & \quad \text{coefficients for Brunger's model} \\
E_{\text{B}} & \quad \text{normal direct illuminance, normalized to } E_{\text{n}} \\
sc, \, \text{b}, \, \text{bl} & \quad \text{coefficients used in ASRC-CIE model} \\
\text{C} & \quad \text{opaque cloud cover} \\
\text{C}, \, \text{c} & \quad \text{normalizing illuminance constant used in Kittler model} \\
\text{c}, \, \text{d} & \quad \text{coefficients used in Perraudau model} \\
E_{\text{ad}} & \quad \text{horizontal diffuse irradiance} \\
E_{\text{deg}} & \quad \text{direct normal extraterrestrial irradiance} \\
E_{\text{eq}} & \quad \text{direct normal irradiance} \\
E_{\text{eqh}} & \quad \text{diffuse illuminance on the horizontal} \\
E_{\text{eqv}} & \quad \text{direct normal extraterrestrial illuminance} \\
E_{\text{eqv}} & \quad \text{direct normal illuminance} \\
f(\gamma) & \quad \text{function of solar incidence used in Perraudau model} \\
f & \quad \text{function of the relative diffuse indicatrix (Kittler)} \\
g(\phi) & \quad \text{function of points elevation used in Perraudau model} \\
G & \quad \text{zenithal gradation function used in Kittler model} \\
G_G & \quad \text{ground reflectance (Kittler)} \\
h(Z) & \quad \text{function of solar zenith angle used in Perraudau model} \\
k & \quad \text{ratio of diffuse to global irradiance} \\
k_s & \quad \text{Solar air mass expression (Makhotkin) used in Kittler model} \\
k_C & \quad \text{clearness index (ratio of global to extraterrestrial irradiance)} \\
k & \quad \text{ratio of diffuse to global illuminance} \\
L_{\text{over}} & \quad \text{lumiance at considered point using CIE standard clear sky} \\
L_{\text{clear}} & \quad \text{lumiance at considered point from CIE clear-turbid expression} \\
L_{\text{inter}} & \quad \text{lumiance at considered point from CIE intermediate sky} \\
L_{\text{over}} & \quad \text{lumiance at considered point from CIE overcast sky} \\
L_{\text{over}} & \quad \text{lumiance at considered point in Harrison model} \\
M & \quad \text{function of } T_z \text{ modulating } f \text{ (Kittler model)} \\
M & \quad \text{sun's air mass} \\
N & \quad \text{function of } T_z \text{ modulating } f \text{ (Kittler model)} \\
N & \quad \text{cloud index used in Perraudau model} \\
P_a & \quad \text{air mass of considered point in Kittler model} \\
P_a & \quad \text{(Makhotkin)} \\
P_a & \quad \text{extinction coefficient for the considered sky element in Kittler model} \\
P_a & \quad \text{function of } Z \text{ and } B \text{ used in Kittler model} \\
P & \quad \text{extinction coefficient in the direction of the sun (Kittler model)} \\
T_z & \quad \text{illumination turbidity coefficient} \\
X & \quad \text{forward elongation of diffusion indicatrix (Kittler model)} \\
Z & \quad \text{enzenith angle} \\
Z & \quad \text{function of } T_z, \, G, \text{ and } X \text{ used in Kittler model} \\
\gamma & \quad \text{angle between the direction of the sun and the considered point} \\
\Delta & \quad \text{sky brightness used in ASRC-CIE model} \\
\epsilon & \quad \text{sky clearness used in ASRC-CIE model} \\
\phi & \quad \text{angular elevation of considered point in the sky} \\
\end{align*}

Appendix: Model-governing equations

\textbf{Brunger's model}

\begin{equation}
L = E_{\text{n}} \left( \left[ a_s + a_s \sin \phi - a_s \exp(a_s \gamma) \right] \left[ \pi (a_s + 2a_s)^3 - 2a_s [1(Z \gamma)] \right] \right)
\end{equation}

where \( L \) is the luminance at the considered point in the sky dome.
\( E_{\text{n}} \) is the horizontal diffuse illuminance.
\( \gamma \) is the angle between the considered point and the sun's position.
\( \phi \) is the angular elevation of the considered point.
\( Z \) is the solar zenith angle (all angles are illustrated in Figure 1).
\( a_s, \, a_s, \, a_s \) and \( a_s \) are four experimentally derived coefficients, function of \( k \), and \( k \) (each function is discrete and consists of a 9-by-9 k-by-k matrix). and

\( I(Z \gamma) \) is a normalizing function equal to

\begin{equation}
I = \left( \pi - I_s - I_s \right)
\end{equation}

with \( I_s = [1 - \exp(a_s \gamma)] /[a_s^2 + 4] \)

\begin{equation}
I_s = 2[1 - \exp(a_s \gamma)]
\end{equation}

\begin{equation}
I_s = \pi a_s \left[ 1 + \exp(-a_s \gamma/2) \right]
\end{equation}

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### Coefficients for the Brunger model:

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<th>$k_r$</th>
<th>0.25</th>
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### Coefficients for Perraudine model:

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\[ I_s = 2Z \sin Z - 0.02\pi \sin(2Z) \] (6)

**Harrison's model**

\[ L_{s,r} = 0.4 + 0.21 Z + 0.27 \sin \phi + 1.45 \exp (-2.41 \gamma) \] (7)

\[ L_{s,t} = [0.128 + 147 \exp(-11.1\gamma)] + 4.28 \cos^2 \gamma \cos Z \times \frac{[1 - \exp(-0.42/\sin \phi)] - [1 - \exp(-0.67/\cos Z)]}{m} \] (8)

The general profile, \( L_{s,t} \), is a linear combination of the two, based on opaque cloud cover, \( C \).

\[ L_{s,t} = C \times L_{s,r} + (1-C) \times L_{s,t} \] (9)

**Kittler's model**

Illuminance turbidity \( T_d \) is obtained from

\[ T_d = (L_{\text{E,n}} - L_{\text{E,n}}) (9.9 + 0.043 m) \div m \] (10)

with \( E_{n,n} \) = direct normal extraterrestrial illuminance

\( E_{n,n} \) = direct normal illuminance at the ground

\( m \) = sun's air mass

The governing equation of the model is

\[ L_s = (C_o, K_s), [(G R_{n,n} Z) + (A - B)(K_s, F - N_s - 3)] \div (K_s - P_s) - 2(A + B)] \] (11)

If the sun's elevation (\( 90^\circ - Z \)) is equal to the considered point's elevation, the equation becomes

\[ L_s = (C_o, K_s) \times [(G R_{n,n} Z) + B \times [0.1 T_d (K_s, F - N_s - 3) - 4]] \] (12)

where \( A = \exp(-P_{m,m} T_d) \)

\( P_{m,m} = 1/9.9 + 0.043 P_s \) (13)

\( P_s = [(\sin^2 \phi + 0.0031465)^{1/2} - \sin \phi] \div 0.001572 \) (14)

\( B = \exp(-S_{m,m} T_d) \)

\( S_{m,m} = 1/9.9 + 0.043 K_s \) (15)

\( K_s = [\cos^2 Z + 0.0031465^{1/2} - \cos Z] \div 0.001572 \) (16)

\( G = 0.8 + G_s, 1 + 1.641 - 0.7 G_s, 3 \sin \phi + A(1 - G_s) 1 + 1.5 \sin \phi \) (17)

with \( G_s = \) ground reflectance (assumed here to be 0.2)

\( R_{n,n} = 1 + B + 1.5 T_d - B \cos Z \) (18)

\( f = 1 - \frac{\exp(-3\gamma)}{0.0099} - M \cos \gamma \) (19)

with \( N = 4.3 T_d, \exp(-0.35T_d) \) (20)

\[ M = 0.71 T_d^{0.5} \] (21)

\[ X_s = 0.115375 \] (22)

\[ Z_s = 1 - \frac{T_d}{0.075 - 0.025 X_s (1 - G_s)} \] (23)

\[ C_s = 7705 \text{ cd/m}^2 \] (24)

**Matsura's model**

\[ L_{s,c} = a_c, L_{c, c,c} + a_c, L_{c, c,c} \] (25)

\[ L_{c, c,c} \] is the CIE clear sky luminance\(^1\) from

\[ L_{c, c,c} \] the CIE intermediate sky luminance\(^2\)

\[ L_{c, c,c} \] is the CIE overcast sky luminance\(^3\)

\( a_c, a, a_c, \) and \( a_a \) are three coefficients depending on \( k_a \) as follows

if \( k_a < 0.5 \)

\[ a_c = 1 \text{ and } a = a_a = 0 \] (26)

if \( k_a \geq 0.3 \) and \( k_a < 0.65 \)

\[ a_c = 1 - (k_a - 0.3) \times 0.35; a = 1 - a_c, \text{ and } a_a = 0 \] (27)

if \( k_a \geq 0.65 \)

\[ a_c = 0, a_a = 1 - (k_a - 0.65) \times 0.35, \text{ and } a_a = 1 - a \] (28)

Note that the analytical formulations of \( L_{c, c,c} \), \( L_{c, c,c} \), and \( L_{c, c,c} \) are provided by Perez, et al. (1990).\(^2\)

**Perradeau's model**

\[ L_s = E_{s,s} f(\gamma) g(\phi) h(Z) \] (29)

where \( E_{s,s} \) is the horizontal diffuse irradiance

\[ f(\gamma) = d_s + e_s \exp(-\gamma) + f_s \cos^2 \gamma \] (30)

\[ g(\phi) = d_s + e_s \cos^2 \phi + f_s \cos \phi \] (31)

\[ h(Z) = d_s + e_s \cos^2 Z \] (32)

where \( d_s, e_s, f_s, d_s, e_s, d_s, e_s, f_s \) are experimentally derived coefficients treated as discrete functions of \( N_p \) defined as \( (1-k_n)(1-k_s) \), where \( k_n \) is the theoretical clear day value of \( n \).

There are five values for each coefficient for conditions ranging from overcast to clear (opposite).
**ASRC-CIE model**

Determination of sky clearness $\varepsilon$ and brightness $\Delta$

\[
e = \frac{(E_{red} + E_{ren})/E_{ref} + 1.041 Z^2}{1 + 1.041 Z^2} \quad (34)
\]

where $E_{ren}$ is the direct normal-incident irradiance, and $\Delta = \frac{E_{red}}{E_{ref}}.$

where $E_{ref}$ is the normal-incident extraterrestrial irradiance.

The model-governing equation is

\[
L_{\varepsilon} = b_\varepsilon L_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon} + b_{\varepsilon\varepsilon} L_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon} - b_\varepsilon L_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon} + b_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon} L_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon} \quad (36)
\]

where $L_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon}$ is a high turbidity form of the CIE clear sky.

$b_\varepsilon,$ $b_{\varepsilon\varepsilon},$ $b_\varepsilon$ and $b_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon}$ are coefficients depending on $\varepsilon$ and $\Delta$ as follows

if $\varepsilon \leq 1.4$

\[
b_\varepsilon = \max \left(0, \min\{1, (\Delta - 0.15)/0.6 + (\varepsilon - 1)/0.4\}\right) \quad (37)
\]

\[
b_{\varepsilon\varepsilon} = 1 - b_\varepsilon \text{ and } b_\varepsilon = b_{\varepsilon\varepsilon} = 0 \quad (38)
\]

if $1.4 < \varepsilon \leq 3$

\[
b_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon} = (\varepsilon - 1.4)/1.6, b_\varepsilon = 1 - b_\varepsilon \text{ and } b_{\varepsilon\varepsilon} = b_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon} = 0 \quad (39)
\]

if $\varepsilon > 3$

\[
b_\varepsilon = \min\{1, (\varepsilon - 3)/3\}, b_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon} = 1 - b_\varepsilon \text{ and } b_\varepsilon = b_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon} = 0 \quad (40)
\]

As above, note that the analytical formulations of $L_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon},$ $L_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon},$ $L_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon}$ and $L_{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon}$ are provided by Perez.\(^2\)